The coordinates of the points of the curve $y = y(x)$ were measured with the aid of two graduated rods. The leveling of the rod Ox was carried out with the aid of an underwater swimming mask on the glass of which a small quantity of water was poured, making the mask into a level gage.

The author would like to thank L.S. Magaziner and V.M. Ryzhik, who assisted in carrying out the measurements and G.I. Ievleva, who performed the computations.

Translated by A.R.R.

ON A STABILITY PROBLEM

(OB ODNOI ZADACHE USTOICHIVOSTI)

PMM Vol.29, № 2, 1965, pp.391-392

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(Received December 2, 1964)

In connection with the frequently passed, at the present time, scientific discussions on the subject of stability of elastic systems with follower forces we have programed and solved the following problem.

A thin elastic bar is executing a uniformly accelerated motion under the action of a follower force, applied at one of its ends.

The differential equation of the elastic line of a homogeneous bar will be

$$EI\frac{\partial^4 y}{\partial x^4} + \frac{\partial}{\partial x}\left[\frac{P}{l}(l-x)\frac{\partial y}{\partial x}\right] + \rho F\frac{\partial^2 y}{\partial t^2} = 0$$

Assuming $y = Xe^{i\Omega t}$ and passing to a nondimensional form we obtain

$$\eta^{\mathrm{IV}} + \beta\,[(1-\zeta)\,\eta]' - \omega^2\eta = 0$$

Here

$$\beta = \frac{Pl^2}{EI}, \quad \omega^2 = \frac{\rho F l^4}{EI}\Omega^2, \quad \zeta = \frac{x}{l}, \quad \eta = \frac{X}{l}$$

The boundary conditions are

$$\eta'' = 0, \quad \eta''' = 0, \quad \text{for } \zeta = 0; \qquad \eta'' = 0, \quad \eta''' = 0 \quad \text{for } \zeta = 1$$

We seek a solution in the form of a series

$$\eta = A_0 + A_1\zeta + A_2\zeta^2 + A_3\zeta^3 + \ldots$$

According to the conditions at the ends

$$A_2 = A_3 = 0, \qquad \Sigma A_n n\,(n-1)\,(n-2) = 0, \qquad \Sigma A_n\, n\,(n-1) = 0 \tag{1}$$

we have for the determination of the terms of the series the recurrence formula

$$A_n = \frac{1}{n(n-1)(n-2)(n-3)}\{\omega^2 A_{n-4} + \beta\,[A_{n-3}(n-3)^2 - A_{n-2}(n-2)(n-3)]\}$$

The constants A_0 and A_1 remain undetermined. They enter linearly in Expressions (1), which can be rewritten in the following form:

$$K_0A_0 + K_1A_1 = 0, \qquad L_0A_0 + L_1A_1 = 0$$

The condition of existence of nonzero solutions is

$$K_0L_1 - K_1L_0 = 0$$

The algorithm of the solution was done on the БЭЦМ (BETSM) computer for 60 terms of the series. For all the values of the parameter β the magnitude of the last term of the series turns out to be smaller than the zero of the computer, i.e. $A_{60} < 10^{-19}$ for $A_0 = 1$, $A_1 = 1$. The results obtained are shown in the Fig.1. The critical state is reached for the value

$$P_* = \frac{109.69EI}{l^2}.$$

A further increase of the load leads to an oscillatory form of motion with increasing amplitude.

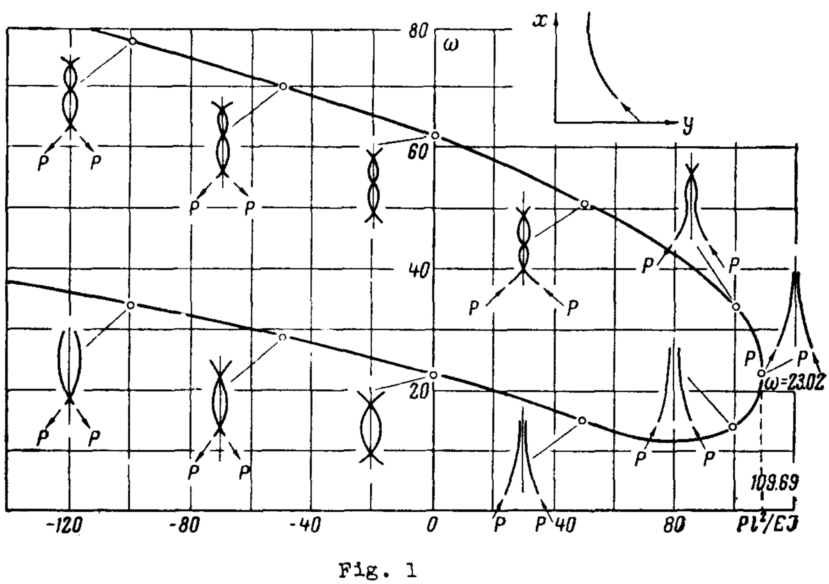

Fig. 1

In the figure are shown the oscillatory shapes of the bar for different values of the force P. An approximate solution, obtained previously by Gopak, gave

$$P_* = \frac{90\,EI}{l^2}$$

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